OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Which term of the A.P. 27, 24, 21, ... is zero?

(a) 5^{th} term **(b) 7^{th} term**

(c) 10^{th} term **(d) 9^{th} term**

[Delhi Gov. SQP 2021]

Ans. *(c) 10^{th} term*

Explanation: Given A.P.

27, 24, 21 ....

Here,

$a = 27, d = -3$

We need to find which term is 0

$\therefore \quad a_n = 0$

$a + (n-1)d = 0$

$27 + (n-1)(-3) = 0$

$27 - 3n + 3 = 0$

$30 - 3n = 0$

$3n = 30$

$$n = \frac{30}{3}$$

$$n = 10$$

$\therefore$ 10th term of A.P. is 0.

2. The list of numbers –10, –6, –2, 2, ... is:

(a) an A.P. with $d = -16$

(b) an A.P. with $d = 4$

(c) an A.P. with $d = -4$

(d) not an A.P. **[NCERT]**

Ans. *(b) an A.P. with $d = 4$*

Explanation: The given list of numbers is

$-10, -6, -2, 2, ...$

Here, $a_1 = -10$, $a_2 = -6$, $a_3 = -2$ and $a_4 = 2$ ...

Since $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

Thus, we can see that the difference between two consecutive terms is same *i.e.* 4.

Hence, the given list forms an A.P. with common difference, $d = 4$.

3. The common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$, ... is:

(a) 1 **(b) $\frac{1}{p}$**

(c) – 1 **(d) $-\frac{1}{p}$** **[CBSE 2020]**

Ans. *(c) –1*

Explanation:

The common difference $= \frac{1-p}{p} - \frac{1}{p}$

$$= \frac{-p}{p} = -1$$

4. The n^{th} term of the A.P. a, $3a$, $5a$, ... is:

(a) na **(b) $(2n-1)a$**

(c) $(2n+1)a$ **(d) $2na$** **[CBSE 2020]**

5. The 11^{th} term of the A.P.: $-5, \frac{-5}{2}, 0, \frac{5}{2}$, ... is:

(a) –20 **(b) 20**

(c) –30 **(d) 30**

[CBSE 2015, 14, 12, NCERT Exemplar]

Ans. *(b) 20*

Explanation: The given A.P. is $-5, \frac{-5}{2}, 0, \frac{5}{2}$ ...

Here, $a = a_1 = -5$, $a_2 = \frac{-5}{2}$, $a_3 = 0$, $a_4 = \frac{5}{2}$

So, $d = \frac{-5}{2} - (-5) = \frac{-5}{2} + 5 = \frac{5}{2}$ $[\because d = a_2 - a_1]$

Now, we know that

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad a_{11} = a + (11-1)d$$

$$= (-5) + (10)\left(\frac{5}{2}\right)$$

$$= -5 + (5)(5) = -5 + 25 = 20$$

$$\Rightarrow \quad a_{11} = 20$$

6. The first four terms of an A.P., whose first term is –2 and the common difference is –2, are:

(a) –2, 0, 2, 4 **(b) –2, 4, –8, 16**

(c) –2, –4, –6, –8 **(d) –2, –4, –8, –16**

[CBSE 2012, NCERT Exemplar]

Ans. *(c) –2, –4, –6, –8*

Explanation: It is given that the first term, $a = -2$ and common difference, $d = -2$.

We know that

$$a_n = a + (n-1)d$$

$\therefore \quad a_1 = -2 + (1-1)(-2) = -2$

$a_2 = -2 + (2-1)(-2) = -2 - 2 = -4$

$a_3 = -2 + (3-1)(-2)$

$= -2 + (2)(-2) = -2 - 4 = -6$

$a_4 = -2 + (4-1)(-2)$

$= -2 + (3)(-2) = -2 - 6 = -8$

Hence, the first four terms of the A.P. are $-2, -4, -6, -8$.

7. The 21^{st} term of an A.P. whose first two terms are –3 and 4, is:

(a) 17 **(b) 137**

(c) 143 **(d) –143** **[NCERT]**

8. The first term of an A.P. is p and the common difference is q, then its 10^{th} term is:

(a) $q + 9p$ **(b) $p - 9q$**

(c) $p + 9q$ **(d) $2p + 9q$**

[CBSE 2020]

Ans. *(c) $p + 9q$*

Explanation:

Here, $a = p$ and $d = q$. Then

$$a_{10} = a + (10-1)d = p + 9q$$

9. If the common difference of an A.P. is 5, then what is the value of $a_{18} - a_{13}$?

(a) 5 **(b) 20**

(c) 25 **(d) 30**

[CBSE 2011, NCERT Exemplar]

10. **Two A.P.s have the same common difference. The first term of one of these is –1 and that of the other is –8. Then the difference between their 4^{th} terms is:**

(a) –1 **(b) –8**

(c) 7 **(d) –9** **[NCERT]**

Ans. *(c) 7*

Explanation: Let d be the common difference of the two A.P.'s and a_1 be the first term of the first A.P. and A_1 be the first term of the second A.P.

Now, $a_1 = -1$, $A_1 = -8$

We know that, $a_n = a + (n-1)d$

$\therefore \quad a_4 = a_1 + (4-1)d$

$= a + 3d = -1 + 3d$

$A_4 = A_1 + (4-1)d$

$= -8 + 3d$

Now, the difference between their 4^{th} terms will be

$|a_4 - A_4| = (-1 + 3d) - (-8 + 3d)$

$= -1 + 3d + 8 - 3d$

$= 7$

Hence, the required difference is 7.

11. **If k, $2k - 1$ and $2k + 1$ are three consecutive terms of an A.P., then the value of k is:**

(a) 2 **(b) 3**

(c) –3 **(d) 5**

Ans. *(b) 3*

Explanation: Here, $k + (2k + 1)$

$= 2(2k - 1)$

i.e. $3k + 1 = 4k - 2$

$\Rightarrow \quad k = 3$

12. **The famous mathematician associated with finding the sum of the first 100 natural numbers is:**

(a) Pythagoras **(b) Newton**

(c) Gauss **(d) Euclid** **[NCERT]**

Ans. *(c) Gauss*

Explanation: Newton is famous for his laws of physics.

Pythagoras is famous for the Pythagorean theorem of a right-angled triangle.

Gauss is the famous mathematician associated with finding the sum of the first 100 natural numbers.

Euclid is most famous for his work in geometry.

13. **If the first term of an A.P. is –5 and the common difference is 2, then the sum of the first 6 terms is:**

(a) 0 **(b) 5**

(c) 6 **(d) 15**

14. **The 11^{th} term of the A.P.: $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$ is:**

(a) $17\sqrt{2}$ **(b) $19\sqrt{2}$**

(c) $21\sqrt{2}$ **(d) $23\sqrt{2}$**

Ans. *(c) $21\sqrt{2}$*

Explanation: The given A.P. is $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

Here, $a = \sqrt{2}$

and, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

Now, 11^{th} term $= a + 10d$

$= \sqrt{2} + 10(2\sqrt{2})$

$= 21\sqrt{2}$

15. **The sum of the first 16 terms of the A.P.: 10, 6, 2, ... is:**

(a) –320 **(b) 320**

(c) –352 **(d) –400**

[CBSE 2012, NCERT Exemplar]

Ans. *(a) –320*

Explanation: The given series of A.P. is 10, 6, 2 ...

Here, the first term, $a = 10$

and common difference, $d = a_2 - a_1$

$= 6 - 10 = -4$

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{16} = \frac{16}{2}[2(10) + (16-1)(-4)]$

$= 8[20 + 15(-4)]$

$= 8[20 - 60]$

$= 8(-40) = -320$

$\Rightarrow \quad S_{16} = -320$

16. **The common difference of an A.P., whose n^{th} term is $a_n = (3n + 7)$, is:**

(a) 3 **(b) 7**

(c) 10 **(d) 6** **[CBSE 2020]**

Ans. *(a) 3*

Explanation: Here,

$a_n = 3n + 7$

$\Rightarrow \quad a_1 = 3 \times 1 + 7 = 10$

and $\quad a_2 = 3 \times 2 + 7 = 13$

So, $\quad d = a_2 - a_1 = 13 - 10 = 3$

17. The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an A.P., is:

(a) – 1 (b) – 2
(c) 1 (d) 2 [CBSE 2020]

18. The number of terms of an A.P. 5, 9, 13, ... 185 is:

(a) 31 (b) 51
(c) 41 (d) 46 [CBSE 2020]

Ans. *(d) 46*

Explanation: The given A.P. is

5, 9, 13, ... 185

Here, $a = 5$, $d = 9 - 5 = 4$ and $l = 185$

We know, last term of an A.P. is given by,

$$l = a + (n - 1)d$$

$$\Rightarrow \quad 185 = 5 + (n - 1)4$$

$$\Rightarrow \quad 180 = (n - 1)4$$

$$\Rightarrow \quad n - 1 = 45$$

$$\Rightarrow \quad n = 46$$

Thus, there are 46 terms in the given A.P.

19. The first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$. Then y is:

(a) –3 (b) 4
(c) 5 (d) 2 [CBSE 2014]

Ans. *(c) 5*

Explanation:

Here, $(3y - 1)$, $(3y + 5)$ and $(5y + 1)$ are in A.P.

Then, $(3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$

$$\Rightarrow \quad 3y + 5 - 3y + 1 = 5y + 1 - 3y - 5$$

$$\Rightarrow \quad 6 = 2y - 4$$

$$\Rightarrow \quad 2y = 10$$

$$\Rightarrow \quad y = 5$$

20. The sum of first 20 odd natural numbers is:

(a) 100 (b) 210
(c) 400 (d) 420 [CBSE 2012]

Ans. *(c) 400*

Explanation:

The first 20 odd natural numbers are:

1, 3, 5, 7, ...

If forms an A.P., with first term, $a = 1$ and common difference, $d = 2$.

We know, $S_n = \frac{n}{2}[2a + (n - 1)d]$

So, sum of first 20 terms is given by,

$$S_{20} = \frac{20}{2}[2 \times 1 + (20 - 1) \times 2]$$

$$= 10[2 + 38]$$

$$= 400$$

21. In an A.P., if $d = -2$, $n = 5$ and $a_n = 0$, then the value of a is:

(a) 10 (b) 5
(c) –8 (d) 8 [CBSE 2020]

22. If the common difference of an A.P. is 3, then the value of $a_{20} - a_{15}$ is:

(a) 5 (b) 3
(c) 15 (d) 20 [CBSE 2011]

23. Ruby was travelling by Rajdhani Express from New Delhi to Sealdah with her friends just before Durga Puja. As all the compartments of the train are interconnected, she and her friends thought of going from one end of the train to the other. She observed that the number of passengers in each coach was not equal. In fact, the difference between the number of passengers in successive coaches was a constant. She thought of making a sequence of number of passengers.

The first term of that A.P. is 5 and the last term is 45. If the sum of all the terms is 400, the number of terms is:

(a) 20 (b) 8
(c) 10 (d) 16

Ans. *(d) 16*

Explanation: Let there be 'n' terms in A.P.

Here, $a = 5$ and $a_n = 45$

Also, $S_n = \frac{n}{2}[a + a_n] = 400$

$$\Rightarrow \quad \frac{n}{2}(5 + 45) = 400$$

$$\Rightarrow \quad n = 16$$

Thus, A.P. has 16 terms.

24. The speedometer of a car displays the instantaneous speed of the car. While driving his car, Soham realised that whenever he pressed the accelerator in such a way that the speed of the car increases uniformly, the speed displayed

increased by a fixed amount. This means that the speed of the car at different time intervals forms an Arithmetic progression.

Let the speed of car form an A.P.: 21, 42, 63, 84, ... then which term is 210?

(a) 9^{th} **(b) 10^{th}**

(c) 11^{th} **(d) 12^{th}**

[Mod. CBSE 2012, Mod. NCERT Exemplar]

Ans. *(b) 10^{th}*

Explanation: The given series is 21, 42, 63, 84, ...

Here, the first term, $a = 21$

and common difference,

$$d = 42 - 21 = 21$$

Let the n^{th} term of the given A.P. be 210.

We know that, $a_n = a + (n - 1)d$

$\Rightarrow \quad 210 = 21 + (n - 1)21$

$\Rightarrow \quad 210 = 21 + 21n - 21$

$\Rightarrow \quad 210 = 21n$

$\Rightarrow \quad n = 10$

Hence, 210 is the 10^{th} term of the given A.P.

25. Prema joined a college as a librarian after completing her degree in library science. She was offered a consolidated monthly salary along with a fixed annual increment. The monthly salary after every year thus formed an arithmetic progression.

Let the three terms of that A.P. be $2x$, $(x + 10)$ and $(3x + 2)$, then the value of x is:

(a) 6 **(b) –6**

(c) 18 **(d) –18**

[Mod. CBSE 2012]

Ans. *(a) 6*

Explanation: Since, $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P.,

$$2(x + 10) = 2x + (3x + 2) \quad [\because 2b = a + c]$$

$\Rightarrow \quad 2x + 20 = 5x + 2$

$\Rightarrow \quad 3x = 18$

$\Rightarrow \quad x = 6$

Concept Applied

If a, b, c are three consecutive terms of an A.P., then

$$b - a = c - b$$

[$\because$ their common difference is same]

$\Rightarrow \quad 2b = a + c$

Solving further, we get $b = \frac{a+c}{2}$

Thus, b is also called the arithmetic mean of a and c.

26. Yoga is a group of physical, mental, and spiritual practices or disciplines which originated in ancient India. Pranayama is an ancient breath technique, it involves controlling the breath in different styles and lengths. Smriti attended a Yoga shivir and was advised to start with a certain number of repetitions in kapalbhati and increase it by a fixed number every week.

She forms an A.P., where $a = 1$, $a_n = 20$ and $S_n = 399$, then n is:

(a) 19 **(b) 21**

(c) 38 **(d) 42**

Fill in the Blanks

27. Fill the two blanks in the sequence 2, ..., 26, ... so that the sequence forms an A.P. **[CBSE SQP 2019]**

Ans. 14, 38 **[CBSE Marking Scheme SQP 2019]**

Explanation: Let the two blanks be x and y, respectively.

So, the A.P. is 2, x, 26, y.

We know, difference between two consecutive terms of an A.P. is same.

$\therefore \quad x - 2 = 26 - x = y - 26$

$\Rightarrow \quad x - 2 = 26 - x$

$\Rightarrow \quad 2x = 26 + 2$

$\Rightarrow \quad 2x = 28$

$\Rightarrow \quad x = 14$

Also, $\quad x - 2 = y - 26$

$\Rightarrow \quad 14 - 2 = y - 26$

$\Rightarrow \quad y = 12 + 26$

$= 38$

$\therefore \quad x = 14$ and $y = 38$

28. The sum of first 16 terms of the A.P. 5, 8, 11, 14, ... is ..... .

Ans. 440

Explanation: Here, first term, $a = 5$

Common difference, $d = 8 - 5 = 3$

Number of terms, $n = 16$

We know, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{16} = \frac{16}{2}[2 \times 5 + (16 - 1) \times 3]$

$= 8 [10 + 45]$

$= 8 \times 55 = 440$

29. If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then the value of a is .....

Ans. $\frac{7}{5}$

Explanation: Given: $\frac{4}{5}$, a, 2 are in A.P.

$\therefore \quad a - \frac{4}{5} = 2 - a$

$\Rightarrow \quad 2a = 2 + \frac{4}{5}$

$\Rightarrow \quad 2a = \frac{14}{5}$

$\Rightarrow \quad a = \frac{7}{5}$

30. If 4, x_1, x_2, x_3, 28 are in A.P., then x_3 = ...

Ans. 22

Explanation: Given, 4, x_1, x_2, x_3, 28 are in A.P.

Let d be its common difference.

Now, first term, $a = 4$

and fifth term, $a_5 = 28$

$\because \quad a_5 = a + (5 - 1)\, d = a + 4d$

$\Rightarrow \quad 28 = 4 + 4d$

$\Rightarrow \quad 4d = 24$

$\Rightarrow \quad d = 6$

Now, $\quad x_3 = a_4 = a + 3d$

$= 4 + 3 \times 6 = 22$

31. If $S_n = 5n^2 + 3n$, then n^{th} term is ..... .

Ans. $10n - 2$

Explanation: We know,

$a_n = S_n - S_{n-1}$

$= 5n^2 + 3n - [5(n - 1)^2 + 3(n - 1)]$

$= 5n^2 + 3n - [5n^2 + 5 - 10n + 3n - 3]$

$= 5n^2 + 3n - [5n^2 - 7n + 2]$

$= 10n - 2$

32. The number of terms of A.P.: 18, 16, 14, ... that make the sum zero, is ..... .

33. Second term of the A.P. if its $S_n = n^2 + 2n$, is ..... .

34. 10^{th} term from end of A.P.: 4, 9, 14, ..., 254 is ..... .

Ans. 209

Explanation:

The given A.P. in reversed form is

254, 249, 244, ..., 14, 9, 4.

Now, 10^{th} term from the end of A.P. 4, 9, 14, ... 254

10^{th} term from the beginning of the A.P. 254, ... 14, 9, 4.

So, for the reversed A.P. 254, 249, ... 14, 9, 4

$a = 254$ and $d = 249 - 254 = -5$

So, $\quad a_{10} = 254 + 9(-5)$

$= 254 - 45 = 209$

Concept Applied

➥ *There are two ways to find n^{th} term from the end of an A.P.*

(1) Using the formula.

$a_n = l - (n - 1)d$, where l is the last term of the A.P.

(2) Reversing the order of the A.P., then calculating n^{th} term from the starting.

Assertion Reason

Direction for questions 35 to 39: In question number 35 to 39, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

35. Assertion (A): **Let the positive numbers a, b, c be in A.P., then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in A.P.**

Reason (R): **If each term of the given A.P. is multiplied by abc, then the resulting sequence is also in A.P.**

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation: Given sequence is, $\frac{1}{bc}, \frac{1}{ab}, \frac{1}{ac}$.

Multiplying each term by abc, we get

$\frac{1}{bc} \times abc = a$

$\frac{1}{ac} \times abc = b$

$\frac{1}{ab} \times abc = c$

Resulting sequence is a, b, c.

Hence, both assertion and reason are true and reason is the correct explanation of the assertion

36. Assertion (A): **Common difference of the A.P. -5, -1, 3, 7, ... is 4.**

Reason (R): **Common difference of the A.P. a, a + d, a + 2d, ... is given by d = 2nd term – 1st term.**

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation : Common difference,

$a_2 - a_1 = a_3 - a_2 = d$.

In the A.P. – 5, – 1, 3, 7, ...

Common difference, d = – 1 – (– 5) = 4.

Hence, both assertion and reason are true and reason is the correct explanation of the assertion.

37. Assertion (A): **Common difference of an A.P. in which $a_{21} - a_7 = 84$ is 14.**

Reason (R): **n^{th} term of AP is given by $a_n = a + (n - 1)d$.**

Ans. *(d) Assertion (A) is false but reason (R) is true*

Explanation : We have,

$$a_n = a + (n - 1)\, d$$

$$a_{21} - a_7 = 84$$

$$= [a + (21 - 1)d] - [a + (7 - 1)d] = 84$$

$$a + 20\, d - a - 6d = 84$$

$$14d = 84$$

$$d = \frac{84}{14} = 6$$

$$d = 6$$

Hence, assertion is false but the reason is true.

38. Assertion (A): **Three consecutive terms 2k + 1, 3k + 3 and 5k – 1 form an A.P. then k is equal to 6.**

Reason (R): **In an A.P. a, a + d, a + 2d, ..., the sum of n terms of the A.P. be $S_n = \frac{n}{2}[2a + (n - 1)d]$**

Ans. *(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation:

For 2k + 1, 3k + 3 and 5k –1 to form an A.P.

$$(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)$$

$$k + 2 = 2k - 4$$

$$2 + 4 = 2k - k = k$$

$$k = 6$$

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Hence, both assertion and reason are true but reason is not the correct explanation of the assertion.

39. Assertion (A): **Sum of first 10 terms of the arithmetic progression –0.5, –1.0, –1.5, ... is –27.5.**

Reason (R): **Sum of n terms of an A.P. is given as $S_n = \frac{n}{2}[2a + (n - 1)d]$ where a = first term, d = common difference.**

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation: If first term of an A.P. is 'a' and its common difference is 'd', then the sum of first n terms of the A.P. is given by,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2(-0.5) + (10 - 1)(-0.5]$$

$$= 5[-1 - 4.5]$$

$$= 5(-5.5) = -27.5$$

Hence, both assertion and reason are true and the reason is the correct explanation of the assertion.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

40. *The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. A total of 27 flags have to be fixed at intervals of every 3 m. All the flags are kept at the position of the middle most flag. Arzoo was given the responsibility of placing the flags on both sides of the middle most flag. Arzoo kept her bag where the flags were stored. She could carry only one flag at a time. So she carries one flag from the middle most point, places the flag at its designated place and comes back to the middle most point. She continues this for all flags on both sides.*

(A) Find the distances covered by Arzoo in placing successive flags on one side of the middle most flag in forms an A.P.

(B) Find the total distance covered by Arzoo in placing all the flags on both sides of the middle most flag.

(C) What is the maximum distance Arzoo travelled carrying a flag?

Ans. (B) The distance covered by Arzoo in placing all the flags on both sides of the middle most flag is found by using the formula for finding sum of first n terms of an A.P. *i.e.*

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

The total distance covered by Arzoo in placing all the flags on both sides of the middle most flag = 2 × (Distance covered by Arzoo in placing 13 flags on one side)

Taking $n = 13$, $a = 6$, $d = 6$, we get

Total distance = $2 \times S_n$

$$= 2 \times \left[\frac{13}{2}\{(2 \times 6 + (13-1)6)\}\right]$$

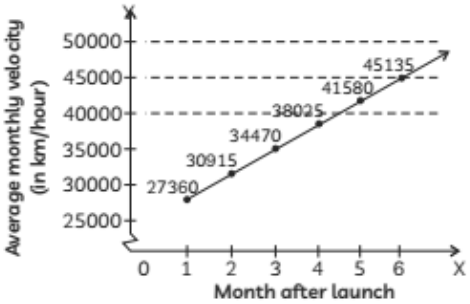
$$= 13 \times [12 + 12 \times 6] = 13 \times 84$$

$$= 1092 \text{ m}$$

41. *In space exploration missions, ion propulsion engine is an efficient way of travelling. The first such engine was used in Deep Space 1 spacecraft. It produced a constant acceleration. Give below are approximate velocities of the engine.*

The initial average velocity of the engine in its first month was 27360 km/hour. When the spacecrafts passed the asteroid, Braille, it reached an average velocity of 55800 km/hour.

Based on the first 6 months of Deep Space 1's monthly average velocity, the following table was created.



Month after launch	Average monthly velocity (in km/hour)
1	27360
2	30915
3	34470
4	38025
5	41580
6	45135

(A) Does the average monthly velocity (in km/hour) form an arithmetic progression? Justify your answer.

(B) The distance travelled by the spacecraft in the first 10 months (or 7300 hours) can be expressed as 7300p km where p is the sum of average monthly velocity for the first 10 months. Find p. Show your work.

(C) The spacecraft passed the comet, Borelly, 15 months after it passed Braille.

Find the average monthly velocity of the spacecraft when it passed Borelly. Show your work.

OR

After how many months did the spacecraft pass Braille? Show your work.

[CBSE Question Bank 2023]

Ans. (A) The first two consecutive average monthly velocity term is 27360 and 30915.

Common difference = 30915 – 27360

= 3555

The next two consecutive average monthly velocity term is 30915 and 34470.

Common difference = 34470 – 30915

= 3555

It concludes that the common difference is same and thus average monthly velocity forms an arithmetic progression.

(B) Given, the distance travelled by the spacecraft in the first 10 months (or 7300 hours) can be expressed as 7300 p.

To find p *i.e.* sum of average monthly velocity for the first 10 months *i.e.*,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow p = S_{10} = \frac{10}{2}[(2 \times 27360) + (10 - 1(3555)]$$

$$= 5[54720 + 31995]$$

$$= 433575$$

Thus, the value of p is 433575 km/hour.

42. *Sumit's father had to undergo knee surgery as he was suffering from a lot of pain and unable to walk properly. The physiotherapist asked him to return to his jogging program slowly. He suggested jogging for 10 minutes daily for the first week and thereafter increase the jogging time by 6 minutes.*

(A) **What will be his jogging time after 8 weeks?**

(a) 46 minutes **(b) 52 minutes**

(c) 58 minutes **(d) 64 minutes**

(B) **In which week will his jogging time be 70 minutes per day?**

(a) 8th week **(b) 10th week**

(c) 11th week **(d) 16th week**

(C) **Which of the following cannot be the n^{th} term of an A.P.?**

(a) $5 + n$ **(b) $2n + 3$**

(c) $5n - 4$ **(d) $2n^2 + 7$**

(D) **The value of k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are the three consecutive terms of an A.P., is:**

(a) 0 **(b) –1**

(c) 1 **(d) 2**

(E) **Which term of the A.P.: 53, 48, 43,... is the first negative term?**

(a) 10th **(b) 12th**

(c) 14th **(d) 16th**

Ans. (B) *(c) 11th week*

Explanation: Here, $a_n = 70$, $a = 10$ and $d = 6$. We have to find n.

We know,

$$a_n = a + (n-1)d$$

$$\Rightarrow 70 = 10 + (n-1)\,6$$

$$\Rightarrow 60 = (n-1)\,6$$

$$\Rightarrow n - 1 = 10$$

Therefore, $n = 11$

His jogging time will be 70 minutes a day in the 11th week.

(D) *(a) 0*

Explanation: Since $(k^2 + 4k + 8)$, $(2k^2 + 3k + 6)$ and $(3k^2 + 4k + 4)$ are in A.P., so

$$(2k^2 + 3k + 6) - (k^2 + 4k + 8) = (3k^2 + 4k + 4) - (2k^2 + 3k + 6)$$

$$\Rightarrow k^2 - k - 2 = k^2 + k - 2$$

$$\Rightarrow -2k = 0$$

Therefore, $k = 0$

(E) *(b) 12th*

Explanation: Let a_n *be the first negative term.*

Then, $a_n < 0$.

Here, $a = 53$, $d = -5$

Therefore,

$$a_n = a + (n-1)d < 0$$

$$\Rightarrow 53 + (n-1)(-5) < 0$$

$$\Rightarrow 53 - 5n + 5 < 0$$

$$\Rightarrow -5n < -58$$

$$\Rightarrow 5n > 58$$

$$n > 11.6$$

Natural number just greater than 11.6 is 12.

$$\therefore n = 12$$

Therefore, the 12th term of the given A.P. is the first negative term.

43. *The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.*

(A) If the first circular row has 30 seats, how many seats will be there in the 10th row?

(B) For 1500 seats in the auditorium, how many rows need to be there?

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

(C) If there were 17 rows in the auditorium, how many seats will be there in the middle row? [CBSE SQP Std. 2022]

Ans. (B) $S_n = \frac{n}{2}[2a+(n-1)d]$

$$1500 = \frac{n}{2}[2\times 30+(n-1)10]$$

$$3000 = 50n + 10n^2$$

$$n^2 + 5n - 300 = 0$$

$$n^2 + 20n - 15n - 300 = 0$$

$$(n + 20)(n - 15) = 0$$

Rejecting the negative value, $n = 15$

OR

No. of seats already put up to the 10th row

$= S_{10}$

$$S_{10} = \frac{10}{2}[2\times 30+(10-1)10]$$

$$= 5(60 + 90) = 750$$

So, the number of seats still required to be put are 1500 – 750 = 750

(C) If no. of rows = 17

then the middle row is the 9th row

$a_8 = a + 8d$

$= 30 + 80$

$= 110$ seats

[CBSE Marking Scheme SQP Std. 2022]

44. *Kaashvi has a piggy bank made of clay where she has been saving part of her pocket money. She saves ₹ 12 in the first month, ₹ 15 in the second month, ₹ 18 in the third month and continues to save in this manner.*

(A) Which of the following is true about the monthly savings of Kaashvi?

(a) It does not form an A.P.

(b) It forms an A.P. with $a = 12$, $d = 3$

(c) It forms an A.P. with $a = 12$, $d = 6$

(d) It forms an A.P. with $a = 12$, $d = -3$

(B) In which month will Kaashvi be able to save ₹ 72?

(a) 18th (b) 20th

(c) 21st (d) 22nd

(C) How much amount did Kaashvi save in the 13th month?

(a) ₹ 57 (b) ₹ 54

(c) ₹ 51 (d) ₹ 48

(D) The third term of an A.P. is 1 and the sixth term is –11. The 15th term of the A.P. is:

(a) – 47 (b) – 51

(c) – 55 (d) – 59

(E) The ratio of the 7th to the 3rd terms of an A.P. is 12 : 5. Then, the ratio of its 13th to 4th terms is:

(a) 3 : 10 (b) 10 : 3

(c) 10 : 5 (d) 3 : 8

Ans. (A) *(b) It forms an A.P. with $a = 12$, $d = 3$*

Explanation: As the monthly savings of Kaashvi are ₹ 12, 15, 18,..., they form an A.P. with $a = 12$ and $d = 3$.

(B) *(c) 21st*

Explanation: To find the month in which Kaashvi will save ₹ 72, we will use the formula for general term of an A.P., *i.e.*

$$a_n = a + (n - 1)d$$

$$\Rightarrow \quad 72 = 12 + (n - 1)3$$

$$\Rightarrow \quad 60 = (n - 1)3$$

$$\Rightarrow \quad n - 1 = 20$$

Therefore, $n = 21$

(D) *(a) –47*

Explanation: It is given that $a_3 = 1$ and $a_6 = -11$.

$$a_3 = 1$$
$$\Rightarrow \quad a + 2d = 1,$$
and $$a_6 = -11$$
$$\Rightarrow \quad a + 5d = -11$$

Solving these equations, we get

$$3d = -12$$

Therefore, $d = -4$

So, $$a = 1 - 2d = 1 + 8 = 9$$

Therefore,

$$a_{15} = 9 + 14 \times (-4)$$
$$= 9 - 56 = -47$$

45. *Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice, it takes him 2 seconds less. He wants to do in 31 seconds.*

(A) Which of the following terms are in A.P. for the given situation?

(a) 51, 53, 55..... **(b) 51, 49, 47 .....**

(c) –51, –53, –55.... **(d) 51, 55, 59....**

(B) What is the minimum number of days?

(a) 10 **(b) 12**

(c) 11 **(d) 9**

(C) Which of the following term is not in the A.P. of the above given situation?

(a) 41 **(b) 30**

(c) 37 **(d) 39**

(D) If n^{th} term of an A.P. is given by $a_n = 2n + 3$, then common difference of an A.P. is:

(a) 2 **(b) 3**

(c) 5 **(d) 1**

(E) The value of x, for which $2x$, $x + 10$, $3x + 2$ are three consecutive terms of an A.P., is:

(a) 6 **(b) –6**

(c) 18 **(d) –18**

[CBSE Question Bank 2021]

Ans. (A) *(b) 51, 49, 47,....*

Explanation: Here, time taken by Veer to cover 200m distance on each passing day is an A.P. with first term $(a) = 51$ and common difference $(d) = -2$ (negative because, time is decreasing).

$$\therefore \quad \text{A.P.} = 51 - 2, 51 - 2(2), ...$$
$$= 51, 49, 47, ...$$

(D) *(a) 2*

Explanation: We know,

Common difference,

$$d = a_n - a_{n-1}$$
$$= 2n + 3 - [2(n - 1) + 3]$$
$$= 2n + 3 - (2n - 2 + 3)$$
$$= 2n + 3 - 2n - 1$$
$$= 2$$

(E) *(a) 6*

Explanation: Since $2x$, $(x + 10)$, $(3x + 2)$ are consecutive terms of an A.P.,

$$\therefore \quad (x + 10) - 2x = (3x + 2) - (x + 10)$$
$$\Rightarrow \quad 10 - x = 2x - 8$$
$$\Rightarrow \quad 18 = 3x$$
$$\Rightarrow \quad x = 6$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

46. In an A.P., if $d = -4$, $n = 7$, $a_n = 4$, then find a.

[CBSE SQP 2020]

Ans.

$$a_n = a + (n - 1)d$$
$$4 = a + 6 \times (-4)$$
$$a = 28$$

[CBSE Marking Scheme SQP 2020]

Explanation: Given, for an A.P.

common difference $d = -4$

number of terms, $n = 7$

n^{th} term, $a_n = 4$

We know, $$a_n = a + (n - 1)d$$

$$\therefore \quad 4 = a + (7 - 1)(-4)$$
$$\Rightarrow \quad 4 = a + (-24)$$
$$\Rightarrow \quad a = 28$$

Hence, the first term of given A.P. is 28.

47. Find the sum of the first 100 natural numbers. [CBSE 2020]

Ans. The list of first 100 natural numbers is. 1, 2, 3, ..., 100, which forms an A.P. with $a = 1$, $d = 1$.

We know, $S_n = \frac{n}{2}[2a + (n-1)d]$

So, $S_{100} = \frac{100}{2}[2\times1+(100-1)(1)]$

$= 50\,[101]$

$= 5050$

48. If the mean of the first n natural numbers is 15, then find n. [CBSE 2020]

Ans. The first n natural numbers are, 1, 2, 3 ..... n.

Their, mean $= \frac{S_n}{n}$,

where $S_n = \frac{n}{2}[2a + (n-1)d]$

Substituting the values, we get

$15 = \frac{1}{2}[2\times1+(n-1)1]$

$\Rightarrow \quad 30 = 2 + n - 1$

$\Rightarrow \quad n = 29$

Hence, the value of n is 29.

Concept Applied

The sum of first n natural numbers or, the sum of the series 1,2, 3, ...n is given by the formula:

$$S = \frac{n(n+1)}{2}$$

49. If in an A.P., $a = 15$, $d = -3$ and $a_n = 0$, then find the value of n. [CBSE 2019]

50. Find the common difference of the Arithmetic Progression (A.P.)

$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$ [CBSE 2019]

Ans. Given: Arithmetic progression (A.P.) is:

$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots, (a \neq 0)$

In the given progression,

$a_1 = \frac{1}{a}$, $a_2 = \frac{3-a}{3a}$ and $a_3 = \frac{3-2a}{3a}$

Common difference,

$d = a_2 - a_1$

$= \frac{3-a}{3a} - \frac{1}{a}$

$= \frac{3-a-3}{3a} = \frac{-a}{3a}$

$= \frac{-1}{3}$

Hence, the common difference of the given A.P. is $\frac{-1}{3}$.

51. Justify whether it is true to say that $-1, -\frac{3}{2}, -2, \frac{5}{2}, \ldots$ form an A.P. as $a_2 - a_1 = a_3 - a_2$.

Ans. No

Explanation: The given series of numbers is $-1, \frac{-3}{2}, -2, \frac{5}{2}, \ldots$

Here, $a_1 = -1$, $a_2 = \frac{-3}{2}$, $a_3 = -2$ and $a_4 = \frac{5}{2}, \ldots$

Difference between two consecutive terms

$a_2 - a_1 = \frac{-3}{2} - (-1) = \frac{-3}{2} + 1 = \frac{-3+2}{2} = \frac{-1}{2}$

$a_3 - a_2 = -2 - \left(\frac{-3}{2}\right) = -2 + \frac{3}{2} = \frac{-4+3}{2} = \frac{-1}{2}$

$a_4 - a_3 = \frac{5}{2} - (-2) = \frac{5}{2} + 2 = \frac{5+4}{2} = \frac{9}{2}$

We can see that $a_2 - a_1 = -\frac{1}{2}$, $a_3 - a_2 = -\frac{1}{2}$,

and $a_4 - a_3 = \frac{9}{2}$.

$\Rightarrow \quad a_2 - a_1 = a_3 - a_2$

but $\quad a_3 - a_2 \neq a_4 - a_3$

Clearly, the difference of two consecutive terms is not the same, hence we can say that the given list of numbers does not form an A.P.

52. How many 2-digit numbers are divisible by 3? [CBSE 2019]

53. Write the n^{th} term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots\ldots$ [CBSE 2017]

Ans. Given, A.P. is $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots\ldots$

Here, first term, $a = \frac{1}{m}$

Common difference,

$d = \frac{1+m}{m} - \frac{1}{m} = \frac{m}{m} = 1$

$\therefore \quad n^{th}$ term, $a_n = a + (n-1)d$

$= \frac{1}{m} + (n-1) \times 1$

$= \frac{1}{m} + n - 1$

$= \frac{1 + mn - m}{m}$

Hence, the n^{th} term of given A.P. is $\frac{mn-m+1}{m}$

54. If the n^{th} term of the A.P. $-1, 4, 9, 14, \ldots$ is 129, find the value of n. [CBSE 2017]

55. Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.
[CBSE 2016]

Ans. Given A.P. is 5, 9, 13, ..., 185,

Since, we have to find the 9th term from the end, we should reverse the A.P. Then, it will become easier to find the 9th term from the starting.

The reversed A.P. is:

185, 181, ..., 13, 9, 5

Now, first term, $a = 185$

Common difference, $d = 181 - 185 = -4$

Then, 9th term of A.P.,

$$a_9 = a + (9-1) \times d \quad [\because a_n = a + (n-1)d]$$
$$= 185 + 8 \times (-4)$$
$$= 185 - 32 = 153$$

Hence, the 9th term from the end of the given A.P. is 153.

56. For the AP: –3, –7, –11, ..., can we directly find $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Ans. *Yes.*

Explanation: The given list of numbers of the A.P. is: –3, –7, –11 ...

We know that, $a_n = a + (n-1)d$

$\Rightarrow \quad a_{30} = a + (30-1)d = a + 29d$

$\Rightarrow \quad a_{20} = a + (20-1)d = a + 19d$

Now, $a_{30} - a_{20} = (a + 29d) - (a + 19d)$

$= a + 29d - a - 19d = 10d$ ...(i)

For the given series,

Common difference, $d = -7 - (-3) = -7 + 3 = -4$

Hence, $a_{30} - a_{20} = 10d = 10(-4) = -40$

[Using eqn. (i)]

Thus, $a_{30} - a_{20} = -40$

57. If the first three terms of an A.P. are b, c and $2b$, then find the ratio of b and c.

Ans. Given: b, c and $2b$ are in A.P

So, c is the arithmetic mean of b and $2b$.

$$\Rightarrow \quad c = \frac{b+2b}{2}$$

$$\Rightarrow \quad c = \frac{3b}{2}$$

$$\Rightarrow \quad \frac{b}{c} = \frac{2}{3}$$

$$\therefore \quad b : c = 2 : 3$$

58. Find the 25th term of the A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2}$.....
[CBSE 2015]

Ans. Here, first term, $a = -5$

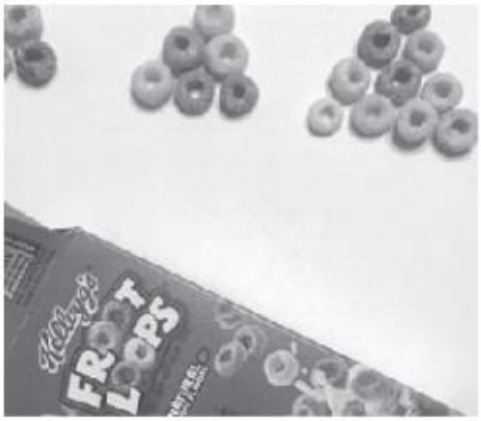
Common difference, $d = \frac{-5}{2} + 5 = \frac{5}{2}$

Then, its 25th term, is

$$a_{25} = a + (n-1)d$$
$$= (-5) + (25-1) \times \frac{5}{2}$$
$$= -5 + 24 \times \frac{5}{2}$$
$$= -5 + 60 = 55$$

59. If the sum of first p terms of an A.P. is $ap^2 + bp$, find its common difference.
[CBSE 2010]

60. Sonam observed her younger brother arrange the froot loops on the table in a certain pattern. She realised that the difference between the number of froot loops between successive patterns was a constant and hence formed an arithmetic progression.



Sonam formed the A.P. as $18, 15\frac{1}{2}, 13, ..., -47$.

Find the number of terms in the A.P.
[Mod. CBSE 2019]

Ans. Given: A.P. is $18, 15\frac{1}{2}, 13, ..., -47$.

Here, first term, $a = 18$

Common difference, $d = \frac{31}{2} - 18 = 13 - \frac{31}{2}$ .....

$$= \frac{-5}{2}$$

last term, $a_n = -47$

We know, $a_n = a + (n-1)d$,

$$\Rightarrow \quad -47 = 18 + (n-1) \times \left(\frac{-5}{2}\right)$$

$$\Rightarrow \quad (n-1) \times \left(\frac{-5}{2}\right) = -65$$

$\Rightarrow \quad (n-1) = 26$

$\Rightarrow \quad n = 27$

Hence, the number of terms in the given A.P. is 27.

61. During the lockdown period, Koyal and her mother decided to start an online bakery business. The initial response was quite encouraging and weekly earnings increased by a fixed amount. The weekly earnings therefore formed an arithmetic progression as 2, 7, 12, 17 ..... .

Find the 16th term of the A.P.

Ans. Here, first term, $a = 2$

and common difference,

$d = 7 - 2 = 5$

We know,

$a_n = a + (n-1)d$

$a_{16} = a + 15d$

$= 2 + 15 \times 5$

$= 2 + 75$

$= 77$

62. Since her childhood days, Rashmi had a very good habit of saving whatever money her parents gave her as pocket money. There was a point when her father put a hold on her pocket-money. At the same time, she started buying two of her favorite chocolates later everyday after school and her daily savings started to decrease by a fixed amount daily.

In how many days, will Rashmi lose her savings completely to 0, if the pattern followed is 27, 24, 21,.....0? [CBSE SQP 2020]

Ans.

$a_n = a + (n-1)d$

$0 = 27 + (n-1)(-3)$

$30 = 3n$

$n = 10$

10^{th}

[CBSE Marking Scheme SQP 2020]

Explanation: Given, A.P. is 27, 24, 21 .....

Here, first term, $a = 27$

common difference, $d = 24 - 27 = -3$

Let, the n^{th} term be zero *i.e.*, $a_n = 0$

Then, $a_n = 0$

$a + (n-1) \times d = 0$

$27 + (n-1) \times (-3) = 0$

$\Rightarrow \quad 27 - 3n + 3 = 0$

$\Rightarrow \quad -3n = -30$

$\Rightarrow \quad n = 10$

Hence, the 10^{th} term of the A.P. is zero.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

63. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P. [CBSE 2020]

Ans. $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ will be in AP, if

$2(a^2+b^2) = (a-b)^2 + (a+b)^2$

$[\because 2b = a + c]$

R.H.S. $= (a-b)^2 + (a+b)^2$

$= a^2 + b^2 - 2ab + a^2 + b^2 + 2ab$

$= 2(a^2+b^2) =$ L.H.S.

Hence, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P.

64. If the 17^{th} term of an A.P. exceeds its 10^{th} term by 7, find the common difference. [CBSE 2019]

65. Determine the A.P. whose third term is 16 and 7^{th} term exceeds the 5^{th} term by 12. [CBSE 2016]

Ans. Let, the first term of an A.P. be 'a' and common difference be 'd'.

Given, $a_3 = 16$

i.e., $a + (3-1)d = 16$

$\Rightarrow \quad a + 2d = 16 \quad ...(i)$

and $\quad a_7 = a_5 + 12$

$\Rightarrow \quad a + (7-1)d = a + (5-1)d + 12$

$\Rightarrow \quad a + 6d = a + 4d + 12$

$\Rightarrow \quad 2d = 12$

$\Rightarrow \quad d = 6$

Put the value of d in equation (i), we get

$a + 2 \times 6 = 16$

$\Rightarrow \quad a = 4$

$\therefore$ The first term of the A.P is 4 and its common difference is 6.

Hence, the required A.P. is 4, 10, 16, 22.....

66. Two A.P.s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. The difference between their 10^{th} terms is the same as the difference between their 21^{st} terms, which is the same as the difference between any two corresponding terms. Why? [NCERT]

67. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21^{st} term ?[CBSE 2019]

Ans. Given: A.P. is 3, 15, 27, 39, ...

Here, the first term, $a = 3$

Common difference, $d = 15 - 3 = 27 - 15 = 12$

Now, 21^{st} term :

$a_{21} = 3 + (21-1) \times 12$

$= 3 + 20 \times 12$

$= 3 + 240 = 243$

Let n^{th} term of the given A.P. be 120 more than its 21^{st} term.

Then, according to the given condition:

$a_n = a_{21} + 120$

$a_n = 243 + 120$

$\Rightarrow \quad a + (n-1)d = 363$

$\Rightarrow \quad 3 + (n-1) \times 12 = 363$

$\Rightarrow \quad (n-1) = \frac{360}{12}$

$\Rightarrow \quad n = 31$

Hence, the 31^{st} term is 120 more than its 21^{st} term.

68. If the first three terms of an A.P. is 15, 13.5, 12. Find the sum of the first 10 terms. [British Council 2022]

Ans. Here, the first term, $a = 15$

Common difference, $d = 13.5 - 15$

$= 12 - 13.5 = -1.5$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{10}{2}[2 \times 15 + (10-1)(-1.5)]$

$= 5[30 - 13.5]$

$= 82.5$

69. Find the sum of first 20 terms of an A.P. whose n^{th} term is given as $a_n = 5 - 2n$ [CBSE Term-2 2022]

70. If seven times the 7^{th} term of an A.P. is equal to eleven times the 11^{th} term, then what will be its 18^{th} term?

71. The 10^{th} term of an A.P. is –4 and its 22^{nd} term is (– 16). Find its 38^{th} term.

Ans. Let, the first term of A.P. be 'a' and its common difference be 'd'.

Now, $\quad a_{10} = -4 \quad$ (given)

$\Rightarrow \quad a + 9d = -4 \quad ...(i)$

and, $\quad a_{22} = -16 \quad$ (given)

$\Rightarrow \quad a + 21d = -16 \quad ...(ii)$

Subtracting equation (ii) from equation (i), we get

$a + 9d = -4$

$a + 21d = -16$

$- \quad - \quad + \quad$

$-12d = 12$

$\Rightarrow \quad d = -1$

Put $d = -1$ in equation (i), we get

$a = -4 + 9 = 5$

Now,

38^{th} term, $a_{38} = a + (38-1)d$

$= 5 + 37(-1)$

$= -32$

Hence, the 38^{th} term of the A.P. is – 32.

72. Find how many integers between 200 and 500 are divisible by 8. [CBSE 2017]

Ans. Integers between 200 and 500 divisible by 8 are 208, 216, 224 ...., 496.

This series forms an A.P., where first term, $a = 8$, common difference, $d = 8$ and last term, $l = 496$.

Let, the number of such integers be 'n'.

Then, n^{th} term = a_n (last term l)

$= a + (n-1)d$

$\Rightarrow \quad 496 = 208 + (n-1) \times 8$

$\Rightarrow \quad (n-1) \times 8 = 496 - 208$

$\Rightarrow \quad (n-1) = \frac{288}{8} = 36$

$\Rightarrow \quad n = 37$

Hence, the number of terms is 37.

73. Determine the A.P. whose third term is 5 and the seventh term is 9.
[CBSE Term-2 2022, CBSE 2017]

74. If the sum of the first 9 terms of an A.P. is equal to the sum of its first 11 terms, then find the sum of its first 20 terms.
[CBSE 2016]

Ans. Let, the first term of the A.P. be 'a' and its common difference be 'd'.

Given, $S_9 = S_{11}$

$\Rightarrow \quad \frac{9}{2}[2a + (9-1)d] = \frac{11}{2}[2a + (11-1)d]$

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad 9[2a + 8d] = 11[2a + 10d]$

$\Rightarrow \quad 18a + 72d = 22a + 110d$

$\Rightarrow \quad 4a = -38d$

$\Rightarrow \quad 2a = -19d \quad ...(i)$

Now, sum of first 20 terms,

$S_{20} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2a + (20-1)d]$

$= 10[-19d + 19d]$ [from (i)]

$= 10 \times 0 = 0$

Hence, the sum of first 20 terms of the A.P. is zero.

75. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both. **[CBSE 2019]**

Ans. Natural numbers between 102 and 998, which are divisible by 2 and 5 both, *i.e.* 10, are

110, 120, 130, ... 990

Clearly, above series is an A.P. with $a = 110$, $d = 120 - 110 = 10$ and $l = 990$.

Let l be the n^{th} term of this A.P.

So, $l = a + (n-1)d$

$\Rightarrow \quad 990 = 110 + (n-1) \times 10$

$\Rightarrow \quad 880 = (n-1)\,10$

$\Rightarrow \quad n - 1 = 88$

$\Rightarrow \quad n = 89$

Hence, there are 89 natural numbers between 102 and 998 which are divisible by both 2 and 5.

76. Which term of the A.P. 17, $16\frac{1}{5}$, $15\frac{2}{5}$, $14\frac{3}{5}$, ... is the first negative term?

77. The common difference between the terms of two A.P.s is same. If the difference between their 50th terms is 100, what is the difference between their 100th terms? **[Diksha]**

Ans. Let a_1 and a_2 be the first terms of two A.P.'s and d be their common difference.

Now, according to the question,

$(a_1 + 49d) - (a_2 + 49d) = 100$

$a_1 - a_2 = 100 \quad ...(i)$

Then, difference between their 100th terms is

$(a_1 + 99d) - (a_2 + 99d) = a_1 - a_2$

$= 100$ [Using (i)]

Hence, the difference between their 100th terms is 100 *i.e.*, same as difference in their 50th terms.

78. For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the n^{th} term (a_n) = 50. Find n and the sum of first n terms (S_n) of the A.P. **[CBSE 2020]**

Ans. Here, $a = 5$, $d = 3$ and $a_n = 50$

We know, $a_n = a + (n-1)d$

$\Rightarrow \quad 5 + 3(n-1) = 50$

$\Rightarrow \quad 3(n-1) = 45$

$\Rightarrow \quad n - 1 = 15$

or $\quad n = 16$

$\therefore$ Number of terms, $n = 16$

Now, $S_n = S_{16} = \frac{16}{2}[2 \times 5 + (16-1)(3)]$

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 8[10 + 45]$

$= 8 \times 55$

$= 440.$

79. If 6 times the 6th term of an A.P. is equal to 9 times the 9th term, show that its 15th term is zero.

Ans. Let a be the first term and d be the common difference of the A.P.

Given, $6a_6 = 9 \times a_9$

Thus, $6(a + 5d) = 9(a + 8d)$

$\Rightarrow 6a + 30d = 9a + 72d$

$\Rightarrow 3a = -42d$

$\Rightarrow a = -14d$

or $a + 14d = 0$ ...(i)

Now, $a_{15} = a + 14d = 0$ [Using (i)]

Hence, 15th term of the A.P. is zero.

80. Find the sum of all 11 terms of an A.P., whose middle term is 30. **[CBSE 2020]**

Ans. Let 'a' be the first term and 'd', be the common difference of the given A.P.

$\because$ Number of terms, $n = 11$ *i.e.* odd

$\therefore$ Middle term $= \left(\frac{11+1}{2}\right)^{th} = 6^{th}$ term

So, the middle term $= a_6 = a + 5d = 30$.

Now, $S_{11} = \frac{11}{2}[2a + 10d]$

$= 11[a + 5d]$

$= 11 \times 30$ [$\because a + 5d = 30$]

$= 330$

81. Find the sum of first 15 multiples of 8. **[CBSE 2017]**

82. Two A.P.'s have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms? **[Diksha]**

Ans. Let a and A be the first terms of the two A.P.s and their common difference be d.

According to question,

$[a + (100 - 1)d] - [A + (100 - 1)d] = 100$

[$\because a_n = a + (n - 1)d$]

$\Rightarrow [a + 99d] - [A + 99d] = 100$

$\Rightarrow a - A = 100$ ...(i)

Now, difference between their 1000th terms is:

$= [a + (1000 - 1)d] - [A + (1000 - 1)d]$

$= a + 999d - A - 999d$

$= a - A$

$= 100$ [Using (i)]

Therefore, difference between their 1000th terms would be equal to 100.

83. Find the 11th term from the last term (towards the first term) of the A.P. 12, 8, 4, ..., – 84. **[CBSE 2020]**

Ans. The given A.P. in the reverse order is

$-84, -80, -76, \ldots\ldots, 4, 8, 12.$

Here, $a = -84, d = 4$

So, 11th term $= a + 10d = -84 + 40 = -44$

84. Solve for x : $1 + 5 + 9 + 13 + \ldots. + x = 1326$ **[CBSE 2020]**

85. If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term. **[CBSE Term-2 SQP 2022]**

Ans.

$7(a + 6d) = 5(a + 4d)$

$\Rightarrow 2a + 22 = 0$

$\Rightarrow a + 11d = 0$

$\Rightarrow t_{12} = 0$

[CBSE Marking Scheme Term-2 SQP 2022]

Let a and d be the first term and the common difference of the A.P., respectively.

Now, $7a_7 = 5a_5$ [Given]

$\Rightarrow 7(a + 6d) = 5(a + 4d)$

$\Rightarrow 7a + 42d = 5a + 20d$

$\Rightarrow 2a = -22d$

or $a = -11d$ ...(i)

So, $a_{12} = a + 11d$

$= -11d + 11d$ [From (i)]

$= 0$

Hence, the 12th term of the A.P. is zero.

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks each]

86. Justify whether it is true to say that the following are the n^{th} terms of an A.P.:

(A) $2n - 3$ **(B) $3n^2 + 5$**

(C) $1 + n + n^2$ **[NCERT]**

87. Find a, b and c such that the following numbers are in A.P.: a, 7, b, 23, c. **[CBSE 2020, 12, NCERT Exemplar]**

Ans. It is given that a, 7, b, 23, c are in A.P.

So, they have a common difference.

i.e., $7 - a = b - 7 = 23 - b = c - 23$

Taking second and third terms, we get

$$b - 7 = 23 - b$$
$$\Rightarrow \quad 2b = 30$$
$$\Rightarrow \quad b = 15$$

Taking first and second terms, we get

$$7 - a = b - 7$$
$$\Rightarrow \quad 7 - a = 15 - 7 \quad [\because b = 15]$$
$$\Rightarrow \quad 7 - a = 8$$
$$\Rightarrow \quad a = -1$$

Taking third and fourth terms, we get

$$23 - b = c - 23$$
$$\Rightarrow \quad 23 - 15 = c - 23 \quad [\because b = 15]$$
$$\Rightarrow \quad 8 = c - 23$$
$$\Rightarrow \quad c = 31$$

Hence, $a = -1$, $b = 15$ and $c = 31$.

88. Determine the A.P. whose 5^{th} term is 19 and the difference of the 8^{th} term from the 13^{th} term is 20.

[CBSE 2011, NCERT Exemplar]

89. Mala saved ₹ 10 in the first week of a year and then increased her weekly savings by ₹ 2. If in the n^{th} week, her weekly savings is ₹ 68, then find the value of n. Also, find her weekly savings in 52^{nd} week of the year.

Ans. Since, savings done by Mala every week increases uniformly by ₹ 2, so her savings form an A.P. with $a = 10$, $d = 2$ and $a_n = 68$.

We know,

$$a_n = a + (n - 1)d$$
$$\Rightarrow \quad 68 = 10 + (n - 1) \times 2$$
$$\Rightarrow \quad 2(n - 1) = 58$$
$$\Rightarrow \quad n - 1 = 29$$
$$\Rightarrow \quad n = 30$$

So, the value of n is 30.

Now,

$$a_{52} = a + 51d$$
$$= 10 + 51 \times 2$$
$$= 10 + 102$$
$$= 112$$

Hence, her savings in the 52^{nd} week is ₹112.

Caution

➥ *Students should learn to differentiate between savings in the n^{th} week and total savings in n weeks. Otherwise, error could occur.*

90. The sum of the first 30 terms of an A.P. is 1920. If the fourth term is 18, find its 11^{th} term. [CBSE 2020]

Ans. Let a be the first term and d be the common difference of the A.P.

Here, number of terms, $n = 30$

Then, $S_{30} = \frac{30}{2}[2a + 29d] = 1920$

or $2a + 29d = 128$ ...(i)

Also, $a_4 = a + 3d = 18$ ...(ii)

On solving equations (i) and (ii), we get

$$d = 4 \text{ and } a = 6$$

Thus, $a_{11} = a + 10d$

$$= 6 + 40 = 46$$

Hence, 11^{th} term of the A.P. is 46.

91. Find the middle term of the A.P. 7, 13, 19, ..., 247.

Ans. Given: A.P. is 7, 13, 19, ... 247.

So, $a = 7$, $d = 13 - 7 = 6$ and $l = 247$

Let n^{th} term be the last term of this A.P.

So, $a_n = l = a + (n - 1)d$

$$\Rightarrow \quad 247 = 7 + (n - 1) \times 6$$
$$\Rightarrow \quad 240 = (n - 1)6$$
$$\Rightarrow \quad n - 1 = 40$$
$$\Rightarrow \quad n = 41$$

Since, the total number of terms in the given A.P. is 41 *i.e.* odd.

$$\therefore \text{ Middle term} = \left(\frac{n+1}{2}\right)^{th} = 21^{st} \text{term}$$

Now, $a_{21} = a + 20d$

$$= 7 + 20 \times 6$$
$$= 7 + 120$$
$$= 127$$

92. Split 207 into three parts such that these are in A.P. and the product of the two smaller parts is 4623. [NCERT]

93. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3? [CBSE 2014, 11, NCERT Exemplar]

Ans. Numbers between 10 and 300 which when divided by 4 leave a remainder 3 are

11, 15, 19, ...299

Clearly, above series is an A.P.

Here, first term, $a = 11$

common difference, $d = 4$

and, last term $a_n = 299$.

Number of terms, $n = ?$

We know that

$$a_n = a + (n-1)d$$

$\Rightarrow a + (n-1)d = 299$

$\Rightarrow 11 + (n-1)(4) = 299$

$\Rightarrow 11 + 4n - 4 = 299$

$\Rightarrow 4n = 292$

$\Rightarrow n = 73$

Hence, the number of such numbers is 73.

94. **Find the sum of the two middle most terms of the A.P.: $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$.**

[CBSE 2010, NCERT Exemplar]

95. **Show that the sum of all terms of an A.P. whose first term is a, the second term is b and the last term is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.** **[CBSE 2020]**

Ans. Given: first term $a_1 = a$, second term, $a_2 = b$ and last term, $l = c$.

So, common difference, $d = a_2 - a_1 = b - a$

Let this A.P. contains n terms.

Then, $l = a + (n-1)d$

$\Rightarrow c = a + (n-1)(b-a)$

$\Rightarrow c - a = (n-1)(b-a)$

$\Rightarrow n - 1 = \frac{c-a}{b-a}$

$\Rightarrow n = \frac{c-a}{b-a} + 1$

$= \frac{c-a+b-a}{b-a}$

$\Rightarrow n = \frac{b+c-2a}{b-a}$ ...(i)

Now, sum of n terms of A.P. is given by,

$$S_n = \frac{n}{2}[a+l]$$

$= \frac{1}{2}\left(\frac{b+c-2a}{b-a}\right)[a+c]$ [Using (i)]

$\Rightarrow S_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

Hence, proved.

96. **If S_n, the sum of first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, then find the A.P.**

Ans. We have, $S_n = 5n^2 + 3n$

For $n = 1$, $S_1 = 5(1)^2 + 3(1) = 8$

For $n = 2$, $S_2 = 5(2)^2 + 3(2) = 26$

Now, $a = S_1 = 8$

and, $S_2 = a_1 + a_2$

$\Rightarrow 26 = 8 + a_2$

$\Rightarrow a_2 = 18$

$\Rightarrow a + d = 18$

$\Rightarrow 8 + d = 18$

$\Rightarrow d = 10$

$\therefore a = 8$ and $d = 10$

So, the A.P. is 8, 18, 28, 38, ...

Related Theory

Alternatively $a_n = S_n - S_{(n-1)}$

Using this formula, we can find the terms of the A.P.

97. **The first term of an A.P. is –5 and the last term is 45. If the sum of the terms of the A.P. is 120, then find the number of terms and the common difference.**

[CBSE 2019, 17, 14, 10, NCERT Exemplar]

98. **Find the middle terms of the A.P. 7, 13, 19 .... 241.**

[Delhi Gov. QB 2022]

Ans. Given A.P. 7, 13, 19, ... 241

$\therefore a = 7, a_n = 241, d = 13 - 7 = 6$

We know that,

$$a_n = a + (n-1)\,d$$

$\therefore 241 = 7 + (n-1)\,6$

$\therefore \frac{241-7}{6} = n - 1$

$\therefore \frac{234}{6} = n - 1$

$\therefore 39 + 1 = n$

$n = 40$

$\therefore \frac{n}{2} = \frac{40}{2} = 20$ ..... (middle term)

$\therefore \frac{n}{2} + 1 = 21$ ..... (middle term)

$\therefore a_{20} = 7 + (20-1)6$

$\therefore a_{20} = 7 + 19 \times 6 = 121$

$\therefore a_{21} = 121 + 6 = 127$

$\therefore$ Middle terms are 121 and 127.

99. **If the sum of the first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms.**

[CBSE 2016, 13, 12]

100. **Find the sum of first 24 terms of an A.P. whose n^{th} term is given by $a_n = 3 + 2n$.**

[CBSE 2017]

Ans. Given, n^{th} term of an A.P., $a_n = 3 + 2n$

First term of A.P., $a_1 = 3 + 2 \times 1 = 5$

Second term of A.P., $a_2 = 3 + 2 \times 2 = 7$

third term of A.P., $a_3 = 3 + 2 \times 3 = 9$

and,

Common difference of A.P., $d = a_2 - a_1 = a_3 - a_2$

$= 7 - 5 = 9 - 7 = 2$

Sum of n terms is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Here, $a = 5, n = 24$ and $d = 2$

$$\therefore \quad S_{24} = \frac{24}{2}[2 \times 5 + (24-1) \times 2]$$

$$= 12[10 + 46]$$

$$= 12 \times 56 = 672$$

Hence the sum of first 24 terms of the A.P. is 672.

101. **If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, find the sum of its first mn terms.**

[CBSE 2017]

102. **Find the sum of n terms of the series $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots\ldots$**

[CBSE 2017]

Ans. Given series is:

$$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots \text{ } n \text{ terms.}$$

$= (4 + 4 + 4 + \ldots n \text{ times})$

$$- \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + n \text{ terms}\right)$$

$$= 4(1 + 1 + 1 + \ldots \text{ upto } n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + \ldots\ldots + \text{upto } n \text{ terms})$$

$$= 4n - \frac{1}{n} \times \frac{n(n \times 1)}{2}$$

$$= 4n - \frac{(n+1)}{2}$$

$$= \frac{8n - n - 1}{2} = \frac{7n - 1}{2}$$

Hence, the sum of the series is $\frac{7n-1}{2}$.

103. **The eighth term of an A.P. is half its second term and the 11^{th} term exceeds one-third of the fourth term by 1. Find the sum of its first 13 terms.**

Ans. Let a be the first term and d be the common difference of the A.P.

Now, $\quad a_8 = \frac{1}{2}a_2$

or, $\quad 2a_8 = a_2$

$\Rightarrow \quad 2(a + 7d) = a + d$

$\Rightarrow \quad a + 13d = 0 \quad \ldots(i)$

Also, $\quad a_{11} - \frac{1}{3}a_4 = 1$

or, $\quad 3a_{11} - a_4 = 3$

$\Rightarrow 3(a + 10d) - (a + 3d) = 3$

$\Rightarrow \quad 2a + 27d = 3 \quad \ldots(ii)$

Solving equations (i) and (ii), we get

$$a = -39, d = 3$$

Now, sum of first 13 terms

$$= S_{13}$$

$$= \frac{13}{2}[2a + (13-1)d]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)]d\right]$$

$$= \frac{13}{2}[2 \times (-39) + 12 \times 3]$$

$$= \frac{13}{2} \times (-42)$$

$$= -273$$

104. **Find the sum of the first 40 positive integers which give a remainder 1 when divided by 6.** **[CBSE 2016]**

105. **Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1^{st} and 4^{th}) to the product of means (2^{nd} and 3^{rd}) is 5 : 6.** **[Diksha]**

Ans. Let the four parts of the A.P. are $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$.

Now, $a - 3d + a - d + a + d + a + 3d = 56$

$\Rightarrow \quad 4a = 56$

$\Rightarrow \quad a = 14$

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$

[Putting $a = 14$]

$$\Rightarrow \quad \frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$\Rightarrow \quad 1176 - 54d^2 = 980 - 5d^2$

$\Rightarrow \quad 49d^2 = 196$

$\Rightarrow \quad d^2 = 4$

$\Rightarrow \quad d = \pm 2$

When $a = 14$ and $d = 2$, then

The four parts are $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$, *i.e.*, 8, 12, 16, 20.

When, $a = 14$ and $d = -2$, then

The four parts are 20, 16, 12 and 8.

Thus, the four parts are 8, 12, 16, 20 or 20, 16, 12, 8.

106. **If the sum of the first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first *n* terms of the A.P.**

[CBSE 2017]

107. **If the sum of first *m* terms of an A.P. is same as the sum of its first *n* terms, show that the sum of its first $(m + n)$ terms is zero.**

[CBSE SQP 2019]

Ans.

$S_m = S_n$

$\Rightarrow \frac{m}{2}[2a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 2a(m - n) + d(m^2 - m - n^2 + n) = 0$

$\Rightarrow \quad (m - n)[2a + (m + n - 1)d] = 0$

or $\quad S_{m+n} = 0$

[CBSE Marking Scheme SQP 2019]

Explanation: Let a be the first term and d be the common difference of the A.P.

Now, according to the question,

$S_m = S_n$

$\Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 2am + m(m - 1)d = 2an + n(n - 1)d$

$\Rightarrow 2am - 2an = (n^2 - n)d - (m^2 - m)d$

$\Rightarrow 2a(m - n) = [(n^2 - m^2) - (n - m)]d$

$\Rightarrow 2a(m - n) = (n - m)[(n + m) - 1]d$

$\Rightarrow -2a = (m + n - 1)d \quad ...(i)$

Now, sum of $(m + n)$ terms is given by,

$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$

$= \frac{m+n}{2}[2a + (-2a)]$ [Using (i)]

$= \frac{m+n}{2} \times 0$

$= 0$

$\Rightarrow \quad S_{m+n} = 0$

Hence, proved.

108. **Among the natural numbers 1 to 49, find a number *x*, such that the sum of numbers preceding it is equal to sum of numbers succeeding it.** **[Diksha]**

Ans. Let, the required number be *x*.

So, the series of natural numbers is:

1, 2, 3, 4, ..., $x - 1$, x, $(x + 1)$, ... 49.

Now, according to the question,

$1 + 2 + 3 + 4 + ... + x - 1 = (x + 1) + ... + 49$

$\Rightarrow \quad S_{x-1} = S_{49} - S_x$

$\Rightarrow \quad \frac{(x-1)x}{2} = \frac{49 \times 50}{2} - \frac{x(x+1)}{2}$

$\Rightarrow \quad x^2 - x = 2450 - x^2 - x$

$\Rightarrow \quad 2x^2 = 2450$

$\Rightarrow \quad x^2 = 1225$

$\Rightarrow \quad x = \pm 35$

But $x \neq -35$, as number are positive.

$\therefore \quad x = 35$

109. **The 14^{th} term of an A.P. is twice its 8^{th} term. If its 6^{th} term is –8, then find the sum of its first 20 terms.** **[CBSE 2015]**

110. **The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.** **[CBSE 2016]**

Ans. Let the required three digits in A.P. of a 3-digit number be $(a - d)$, a, $(a + d)$ respectively.

Now, $\quad a - d + a + a + d = 15$

$\Rightarrow \quad 3a = 15$

$\Rightarrow \quad a = 5$

According to question, number is:

$100(a - d) + 10a + a + d = 111a - 99d$

Number, on reversing the digits, is:

$100(a + d) + 10a + a - d = 111a + 99d$

Now, as per given condition in question,

$(111a - 99d) - (111a + 99d) = 594$

$\Rightarrow \quad -198d = 594$

$\Rightarrow \quad d = -3$

So, digits of number are $[5 - (-3), 5, (5 + (-3)]$ *i.e.* 8, 5, 2

Hence, the required number is 852.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

111. A sum of ₹ 4,250 is to be used to give 10 cash prizes to students of a school for their overall academic performance. If each prize is ₹ 50 less than its preceding prize, find the value of each of the prizes. [CBSE 2017]

Ans. Let the value of first most expensive prize be ₹ a.

Then, according to the given condition, prizes are a, $a - 50$, $a - 100$, $a - 150$ .....

The given series forms an A.P., with a common difference of (– 50).

Here, first term = a

Common difference $d = -50$

Number of terms, $n = 10$

and, sum of 10 terms, $S_{10} =$ ₹ 4,250

By formula, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \quad S_{10} = \frac{10}{2}[2 \times a + (10-1) \times (-50)]$$

$$\Rightarrow \quad 4250 = 5(2a - 450)$$

$$\Rightarrow \quad 850 = 2a - 450$$

$$\Rightarrow \quad a = \frac{1300}{2} = ₹\ 650$$

Hence, the value of the prizes are : ₹ 650, ₹ 600, ₹ 550, ₹ 500, ₹ 450, ₹ 400, ₹ 350, ₹ 300, ₹ 250, ₹ 200.

112. A child puts one five-rupee coin of her savings in the piggy bank on the first day. She increases her saving by one five-rupee coin daily. If the piggy bank can hold 190 coins of five rupees in all, find the number of days she can continue to put the five-rupee coins into it and find the total money she saved.

Write your views on the habit of saving.

[CBSE 2017]

113. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle term is 7 : 15. Find the numbers. [CBSE 2020]

114. Solve for x : $1 + 4 + 7 + 10 + ... + x = 287$. [CBSE 2020]

115. The sum of the first 5 terms of an A.P. and the sum of the first 7 terms of the same A.P. is 167. If the sum of the first 10 terms of this A.P. is 235, find the sum of its first 20 terms. [NCERT Exemplar]

Ans. Let a be the first term, d be the common difference and n be the total number of terms of the A.P.

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

∴ Sum of first five terms,

$$S_5 = \frac{5}{2}[2a + (5-1)d]$$

$$= \frac{5}{2}[2a + 4d]$$

$$\Rightarrow \quad S_5 = 5[a + 2d]$$

$$\Rightarrow \quad S_5 = 5a + 10d \qquad ...(i)$$

And, sum of first seven terms,

$$S_7 = \frac{7}{2}[2a + (7-1)d]$$

$$= \frac{7}{2}[2a + 6d]$$

$$\Rightarrow \quad S_7 = 7[a + 3d]$$

$$\Rightarrow \quad S_7 = 7a + 21d \qquad ...(ii)$$

Now, by given condition

$$S_5 + S_7 = 167$$

$$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$$

[using equation (i) & equation (ii)]

$$\Rightarrow \quad 12a + 31d = 167 \qquad ...(iii)$$

Also, it is given that sum of first 10 terms of this A.P. is 235.

$$\Rightarrow \quad S_{10} = 235$$

$$\Rightarrow \quad \frac{10}{2}[2a + (10-1)d] = 235$$

$$\Rightarrow \quad 5[2a + 9d] = 235$$

$$\Rightarrow \quad 2a + 9d = \frac{235}{5}$$

$$\Rightarrow \quad 2a + 9d = 47 \qquad ...(iv)$$

Multiplying equation (iv) by 6 and then subtracting it from equation (iii), we get

$$(12a + 31d) - (12a + 54d) = 167 - 282$$
$$\Rightarrow \quad -23d = -115$$
$$\Rightarrow \quad d = 5$$

Putting the value of d in equation (iv), we get

$$2a + 9(5) = 47$$
$$\Rightarrow \quad 2a + 45 = 47$$
$$\Rightarrow \quad 2a = 2$$
$$\Rightarrow \quad a = 1$$

Sum of first 20 terms of this A.P.,

$$S_{20} = \frac{20}{2}[2a + (20 - 1)d]$$
$$= 10[2(1) + 19(5)]$$
$$= 10[2 + 95]$$
$$= 10 \times 97$$
$$= 970$$

Hence, the required sum of first 20 terms is 970.

116. **Find the:**

(A) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5. [CBSE 2014]

(B) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

(C) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[Hint (C): These numbers will be: multiple of 2 + multiple of 5 – multiples of 2 as well as of 5] [NCERT]

117. **An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P.**

Ans. Let a be the first term, d be the common difference and n be the total number of terms of the A.P.

It is given that

Total number of terms, $n = 37$

Since n is odd, therefore

Middle most term $= \left(\frac{n+1}{2}\right)^{th}$ term $= 19^{th}$ term

So, 3 middle most terms $= 18^{th}$, 19^{th} and 20^{th} terms

Now, according to the question

$$a_{18} + a_{19} + a_{20} = 225$$

We know that, $a_n = a + (n - 1)\,d$

$$\Rightarrow (a + 17d) + (a + 18d) + (a + 19d) = 225$$
$$\Rightarrow \quad 3a + 54d = 225$$
$$\Rightarrow \quad a + 18d = 75 \quad \text{...(i)}$$

Also, it is given that sum of last 3 terms = 429

$$\Rightarrow \quad a_{35} + a_{36} + a_{37} = 429$$
$$\Rightarrow \quad (a + 34d) + (a + 35d) + (a + 36d) = 429$$
$$\Rightarrow \quad 3a + 105d = 429$$
$$\Rightarrow \quad a + 35d = 143 \quad \text{...(ii)}$$

Subtracting equation (i) from equation (ii), we get

$$(a + 35d) - (a + 18d) = 143 - 75$$
$$a + 35d - a - 18d = 68$$
$$\Rightarrow \quad 17d = 68$$
$$\Rightarrow \quad d = 4$$

Putting the value of d in equation (i), we get

$$a + 18(4) = 75$$
$$\Rightarrow \quad a + 72 = 75$$
$$\Rightarrow \quad a = 3$$

$\therefore$ Required A.P. is:

$a, a + d, a + 2d, a + 3d, \ldots$

i.e. $3, 3 + 4, 3 + 2(4), 3 + 3(4), \ldots$

i.e. $3, 7, 11, 15, \ldots$

118. **If the sum of the first 'p' terms of an A.P. is 'q' and sum of the first 'q' terms is 'p'; then show that the sum of the first $(p + q)$ terms is $\{-(p + q)\}$. [CBSE 2019]**

119. **Which term of the Arithmetic Progression – 7, – 12, – 17, – 22, ... will be –82? Is –100 any term of the A.P.? Give reason for your answer. [CBSE 2019]**

120. **How many terms of the arithmetic progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.**

Ans. The arithmetic progression is 45, 39, 33, ...

Here, $a = 45$

$d = 39 - 45 = 33 - 39$

$= -6$

and, $S = 180$

Let the sum of first n terms of the given A.P. be 180.

So, $S_n = 180$

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad 180 = \frac{n}{2}[2 \times 45 + (n - 1)(-6)]$

$\Rightarrow \quad 360 = 96n - 6n^2$

$\Rightarrow \quad 6n^2 - 96n + 360 = 0$

$\Rightarrow \quad 6[n^2 - 16n + 60] = 0$

$\Rightarrow \quad 6[n^2 - 10n - 6n + 60] = 0$

$\Rightarrow \quad 6[n(n-10) - 6(n-10)] = 0$

$\Rightarrow \quad 6[(n-6)(n-10)] = 0$

$\Rightarrow \quad n = 6, 10$

$\therefore$ Sum of a_7, a_8, a_9 and a_{10} terms = 0

Hence, on either adding 6 terms or 10 terms, we get a total of 180.

121. **Ramkali would require ₹ 5000 for getting her daughter admitted in a school after a year. She saved ₹ 150 in the first month and increased her monthly saving by ₹ 50 every month. Find, if she will be able to arrange the required money after 12 months. Which value is reflected in her efforts?** **[CBSE 2019, 15]**

122. **The sum of the 4th and the 8th term of an A.P. is 24 and the sum of the 6th and the 10th term is 44. Find the sum of the first 10 terms of the A.P.** **[CBSE 2017]**

123. **If the ratio of the 11th term of an A.P. to its 18th term is 2 : 3, find the ratio of the sum of the first five terms to the sum of its first 10 terms.** **[CBSE 2013]**

Ans. Let, the first term of A.P. be 'a' and its common difference be 'd'.

Then, 11th term of A.P., $a_{11} = a + 10d$

and, 18th term of A.P., $a_{18} = a + 17d$

So, $\frac{a+10d}{a+17d} = \frac{2}{3}$ (given)

$\Rightarrow \quad 3(a + 10d) = 2(a + 17d)$

$\Rightarrow \quad 3a + 30d = 2a + 34d$

$\Rightarrow \quad a = 4d$ ...(i)

Now, sum of first five terms,

$$S_5 = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{5}{2}[2 \times 4d + 4 \times d]$$

[from (i)]

$$= \frac{5}{2} \times 12d = 30d$$

Sum of first 10 terms,

$$S_{10} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2 \times 4d + 9 \times d]$$

$$= 5(8d + 9d)$$

$$= 85d$$

$\therefore \quad \frac{S_5}{S_{10}} = \frac{30d}{85d} = \frac{6}{17}$

Hence, the required ratio is 6 : 17.

124. **The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.** **[CBSE 2017]**

125. **Solve for x :**

$-4 + (-1) + 2 + \ldots + x = 437$.

[NCERT Exemplar]

126. **The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b and c respectively.**

Show that $a(q - r) + b(r - p) + c(p - q) = 0$

[CBSE SQP 2020]

Ans. Let A be the first term and D be the common difference of A.P.

$T_p = a = A + (p-1)D$

$= (A - D) + pD$ ...(1)

$T_q = b = A + (q-1)D$

$= (A - D) + qD$ ...(2)

$T_r = c = A + (r-1)D$

$= (A - D) + rD$ ...(3)

Here we have got two unknowns A and D which are to be eliminated. We multiply (1),(2) and (3) by $q - r$, $r - p$ and $p - q$ respectively and add :

$a(q - r) = (A - D)(q - r) + Dp(q - r)$

$b(r - p) = (A - D)(r - p) + Dq(r - p)$

$c(p - q) = (A - D)(p - q) + Dr(p - q)$

$a(q - r) + b(r - p) + c(p - q)$

$= (A - D)[q - r + r - p + p - q]$

$+ D[p(q - r)$

$+ q(r - p) + r(p - q)]$

$= (A - D)(0) + D[pq - pr$

$+ qr - pq + rp - rq)$

$= 0.$

[CBSE Marking Scheme SQP 2020]

127. **Yasmeen saves ₹ 32 during the first month, ₹ 36 in the second month and ₹ 40 in the third month. If she continues to save in this manner, in how many months will she save ₹ 2000?** **[CBSE 2016, 15, 12]**

128. A thief runs with a uniform speed of 100 m/minute. After one minute, a policeman ran after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief?

[CBSE 2016]

Ans. Let, the total time taken to catch the thief be 'n' minutes.

$\because$ Speed of thief = 100 m/min.,

$\therefore$ Total distance covered by the thief = 100 m

Now, the speed of the policeman in the first minute is 100 m/min, in the 2^{nd} minute is 110 m/min, in the 3^{rd} minute is 120 m/min and so on.

Then, the speed forms an A.P. with a constant increasing speed of 10 m/min. thus, the series is :

100, 110, 120, 130, ...

As the policeman starts after a minute, so time taken by the policeman to catch the thief is $(n - 1)$ minutes.

$\therefore$ Total distance covered by the policeman

$= 100 + 110 + 120 + \_ (n - 1)$ terms

$$\therefore \quad 100n = \frac{n-1}{2}[200 + (n-2) \times 10]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)]d\right]$$

$$\Rightarrow \quad 200n = (n-1)[200 + 10n - 20]$$

$$\Rightarrow \quad 200n = (n-1)(180 + 10n)$$

$$\Rightarrow \quad 200n = 180n - 180 + 10n^2 - 10n$$

$$\Rightarrow \quad 10n^2 - 30n - 180 = 0$$

$$\Rightarrow \quad n^2 - 3n - 18 = 0$$

$$\Rightarrow \quad (n-6)(n+3) = 0$$

$$\Rightarrow \quad n = 6$$

$[\because n = -3$, is not possible$]$

Hence, the policeman takes $(n - 1) = 5$ minutes to catch the thief.